Noise in Analog CMOS ICs

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March 2013

Noise = Unwanted Signal

- Intrinsic (inherent) noise:
  - generated by random physical effects in the devices.
- Interference (environmental) noise:
  - coupled from outside into the circuit considered.
- Switching noise:
  - charge injection, clock feedthrough, digital noise.
- Mismatch effects:
  - offset, gain, nonuniformity, ADC/DAC nonlinearity errors.
- Quantization (truncation) “noise”:
  - in internal ADCs, DSP operations.

Purpose and Topics Covered

- Purpose: Fast Estimation of Noise in Analog Integrated Circuits Before Computer Simulation. Topics Covered:
- The Characterization of Continuous and Sampled Noise;
- Thermal Noise in Opamps;
- Thermal Noise in Feedback Amplifiers;
- Noise in an SC Branch;
- Noise Calculation in Simple SC Stages;
- Sampled Noise in Opamps.

Characterization of Continuous-Time Random Noise – (1)

- Noise $\boldsymbol{x(t)}$ – e.g., voltage. Must be *stationary* as well as *mean and variance ergodic.*
- *Average power* defined as mean square value of $\boldsymbol{x(t)}$:

$$P_{av} = \lim_{T\to\infty}\left\{\frac{1}{T}\int_0^T x^2(t)dt\right\} = E\left\{x^2(t)\right\}$$

- For uncorrelated zero-mean noises

$$E\{(x_1 + x_2)^2\} = E\{x_1^2\} + E\{x_2^2\}$$

- *Power spectral density (PSD)* $\boldsymbol{S_x(f)}$ of $\boldsymbol{x(t)}$: $\boldsymbol{P_{av}}$ contained in a 1Hz BW at $\boldsymbol{f}$. Hence $d\boldsymbol{P_{av}} = \boldsymbol{S_x(f).df}$. Measured in V^2/Hz. Even function of $\boldsymbol{f}$.

- Filtered noise: if the filter has a voltage transfer function H(s), the output noise PSD is $|H(j2\pi f)^2|S_x(f)$.

Characterization of Continuous-Time Random Noise – (2)

- Average power in $f_1 < f < f_2$:

$$P_{f_1,f_2} = \int_{f_1}^{f_2} S_x(f)\,df$$

Here, S_x is regarded as a one-sided PSD.

Hence, $P_{0,\infty} = P_{av}$. So white noise has infinite power!? NO. Quantum effects occur when $h.f \sim kT$. Here, h is the Planck constant. This occurs at tens of THz; parasitics will limit noise much before this!

- Amplitude distribution: *probability density function(PDF)* $p_x(x)$. $p_x(x_1)\Delta x$: probability of $x_1 < x < x_1+\Delta x$ occurring. E.g., $p_x(q)=1/LSB$ for quantization noise $q(t)$ if $|q| < LSB/2$, 0 otherwise. Gaussian for thermal noise, often uniform for quantization noise.

Characterization of Sampled-Data Random Noise

- Noise $x(n)$ – e.g., voltage samples.
- Ave. power: $$P_{av} = E\left[\lim_{N\to\infty}\left\{\frac{1}{N+1}\sum_{n=0}^{N}\left|x(n)^2\right|\right\}\right]$$

  P_{av} is the mean square value of $x(n)$.
- For sampled noise, P_{av} remains invariant!
- Autocorrelation sequence of $x(n)$ – shows periodicities!

$$r_x(k) = E\{x(n)\cdot x(n-k)\}$$
$$r_x(0) = [P_{av} \quad of \quad x(n)] = E\{|x(n)|^2\}$$

- PSD of $x(n)$: $S_x(f) = \mathcal{F}[r_x(k)]$. Periodic even function of f with a period f_c. Real-valued, non-negative.
- Power in $f_1 < f < f_2$: $P_{f_1,f_2} = \int_{f_1}^{f_2} S_x(f)df$

$$P_{av} = \int_0^{f_c/2} S_x(f)df = P_{0,f_x/2} = E\{|x(n)|^2\}$$

Sampled noise has finite power!

- PDF of $x(n)$: $p_x(x)$, defined as before.

Thermal Noise – (1)

Due to the random motion of carriers with the MS velocity ∝ T.

Dominates over shot noise for high carrier density but low drift velocity, occurring, e.g., in a MOSFET channel.

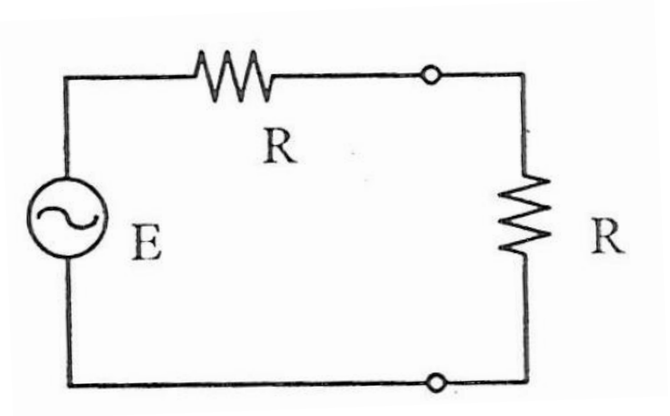
Mean value of velocity, noise V, I is 0.

The power spectral density of thermal noise is $PSD = kT$. In a resistive voltage source the maximum available noise power is hence

$$\frac{\overline{E^2}}{4 \cdot R} = k \cdot T \cdot BW$$

giving

$$\overline{E^2} = 4 \cdot k \cdot T \cdot BW \cdot R$$



k: Boltzmann constant, 1.38x10^-23 J/K

Bode's Noise Theorem

- Passive RLC switched circuit:

$$\overline{v_o^2} = kT\left(\frac{1}{C_\infty} - \frac{1}{C_0}\right). \qquad (7)$$

PAVAN: ALTERNATIVE APPROACH TO BODE'S NOISE THEOREM

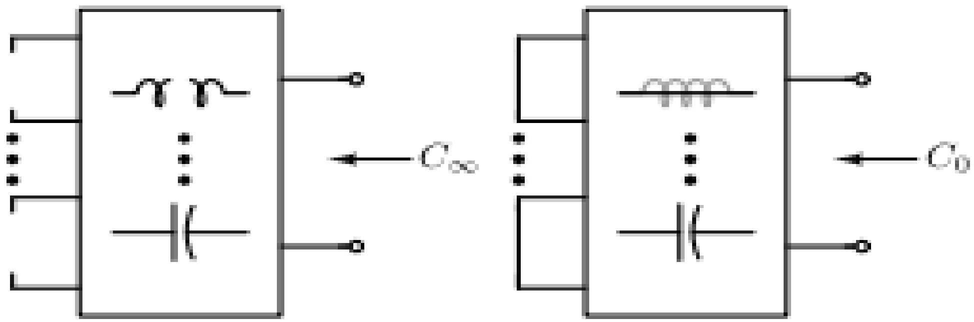
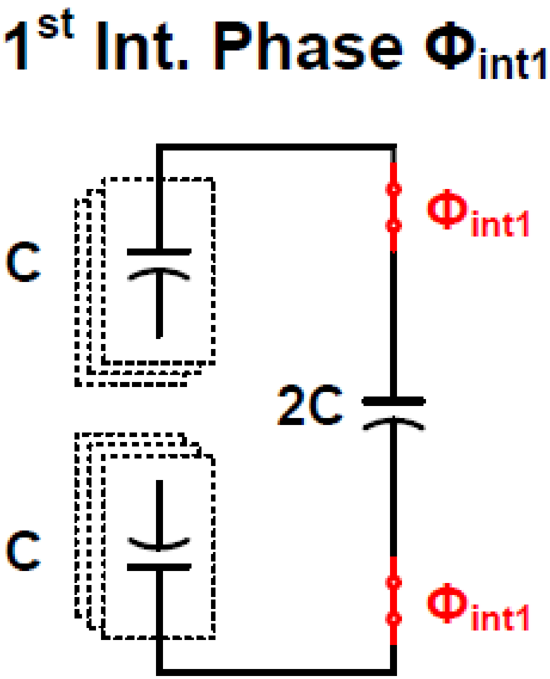
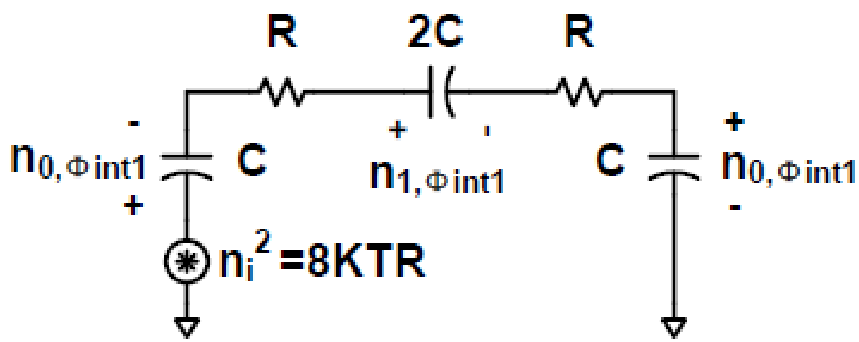


Fig. 3. Physical interpretation of C_∞ and C_0 : C_∞ is the looking-in capacitance of the network when all resistors and inductors are opened, and C_0 is the capacitance looking in when all the resistors and inductors are shorted.

Bode's Noise Theorem

- Example:



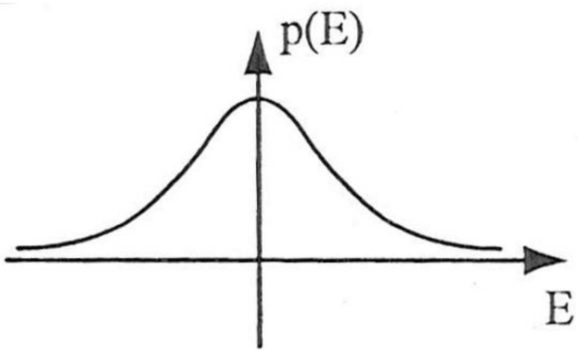


$$\frac{n_{1,\phi int1}}{n_i} = \frac{0.2}{1 + \frac{s}{1/0.8RC}}, \qquad \frac{n_{0,\phi int1}}{n_i} = \frac{0.4}{1 + \frac{s}{1/0.8RC}}$$

- $n_{1,\phi int1}^2 = 0.2^2 \frac{1}{2\pi(0.8RC)} \frac{\pi}{2} 8kTR = \frac{0.1kT}{C}$
- $n_{0,\phi int1}^2 = \frac{0.4kT}{C}$

Thermal Noise – (2)

The probability density function of the noise amplitude follows a Gaussian distribution



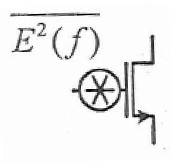
_2
E

$$p(E) = \frac{e^{-E^2/\overline{E^2}}}{\sqrt{\overline{E^2}\cdot 2\pi}}$$

Here, $\overline{E^2}$ is the MS value of ***E***.

In a MOSFET, if it operates in the triode region, R=rds can be used in the drain-source branch.

In the active region, averaging over the tapered channel, ***R=(3/2)/g***$_m$ results. The equivalent circuit is



$$\overline{E^2(f)} = \frac{8kT}{g_m}(PSD)$$

8/3

Noise Bandwidth

Let a white noise ***x(t)*** with a ***PSD*** S_x enter an LPF with a transfer function:

$$H(s) = \frac{G_0}{s/\omega_{3-dB} + 1}$$

Here G_0 is the dc gain, and ω_{3-dB} is the 3-dB BW of the filter. The MS value of the output noise will be the integral of $|H|^2 \cdot S_x$, which gives

$$\overline{x^2} = \frac{\omega_{3-dB}}{4} G_0^2 \cdot S_x$$

Assume now that ***x(t)*** is entered into an ideal LPF with the gain function:

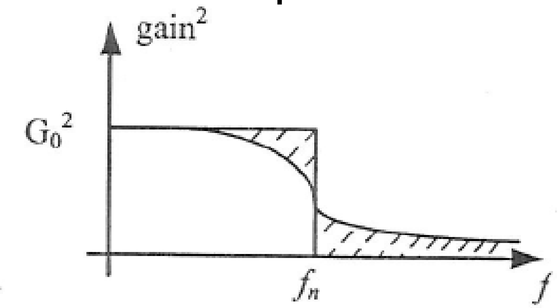
$$|H| = G_0 \qquad \text{if } f < f_n$$

and 0 if $f > f_n$. The MS value of the output will then be:

$$\overline{x^2} = f_n \cdot G_0^2 \cdot S_x$$

Equating the RHSs reveals that the two filters will have equivalent noise transfer properties if

$$f_n = \frac{\omega_{3-dB}}{4} = \frac{\pi}{2} f_{3-dB}$$



Here, f_n is the *noise bandwidth* of the LPF.

Analysis of 1/f Noise in Switched MOSFET Circuits

IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—II: ANALOG AND DIGITAL SIGNAL PROCESSING, VOL. 48, NO. 2, FEBRUARY 2001 151

Analysis of $1/f$ Noise in Switched MOSFET Circuits

Hui Tian and Abbas El Gamal, *Fellow, IEEE*

Abstract—Analysis of $1/f$ noise in MOSFET circuits is typically performed in the frequency domain using the standard stationary $1/f$ noise model. Recent experimental results, however, have shown that the estimates using this model can be quite inaccurate especially for switched circuits. In the case of a periodically switched transistor, measured $1/f$ noise power spectral density (psd) was shown to be significantly lower than the estimate using the standard $1/f$ noise model. For a ring oscillator, measured $1/f$-induced phase noise psd was shown to be significantly lower than the estimate using the standard $1/f$ noise model. For a source follower reset circuit, measured $1/f$ noise power was also shown to be lower than the estimate using the standard $1/f$ model. In analyzing noise in the follower reset circuit using frequency-domain analysis, a low cutoff frequency that is inversely proportional to the circuit on-time is assumed. The choice of this low cutoff frequency is quite arbitrary and can cause significant inaccuracy in estimating noise power. Moreover, during reset, the circuit is not in steady state, and thus frequency-domain analysis does not apply.

This paper proposes a nonstationary extension of the standard $1/f$ noise model, which allows us to analyze $1/f$ noise in switched MOSFET circuits more accurately. Using our model, we analyze noise for the three aforementioned switched circuit examples and obtain results that are consistent with the reported measurements.

Index Terms—$1/f$ noise, CMOS image sensor, nonstationary noise model, periodically switched circuits, phase noise, ring oscillator, time-domain noise analysis.

especially for switched circuits. An important class of such circuits is periodically switched circuits, which are widely used in RF applications, such as switched capacitor networks, modulators and demodulators, and frequency converters. In the simplest case of a periodically switched transistor, it was shown that the measured drain voltage $1/f$ noise power spectral density (psd) [5]–[7] is much lower than the estimate using the standard $1/f$ noise model. Another example that has recently been receiving much attention is $1/f$-induced phase noise in CMOS oscillators [8]–[10]. Unlike the amplitude fluctuations, which can be practically eliminated by applying limiters to the output signal, phase noise cannot be reduced in the same manner. As a result, phase noise limits the available channels in wireless communication. Recent measurements [7] show that the $1/f$-induced phase noise psd in ring oscillators is much lower than the estimate using the standard $1/f$ noise model.

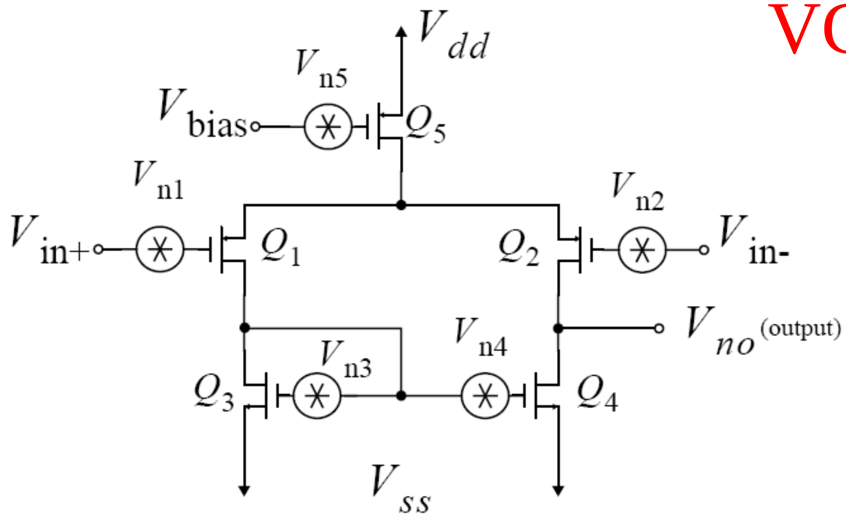
Yet another example of a switched circuit is the source follower reset circuit, which is often used in the output stage of a charge-coupled device (CCD) image sensor [11] and the pixel circuit of a CMOS active pixel sensor (APS) [12]. To find the output noise power due to $1/f$ noise, frequency-domain analysis is typically performed using the standard $1/f$ noise model. A low cutoff frequency f_L that is inversely proportional to the

- ***1/f*** noise can be represented as threshold voltage variation.
- If the switch is part of an SC branch, it is unimportant. In opamps,it is critical!
- In a chopper or modulator circuit, it may be very important. See the TCAS paper shown.

Thermal Op-Amp Noise – (1)

Simple op-amp input stage [3]:

VCCS model used



With device noise PSD: $\hat{V}_{ni}^2 = (8/3)kT/g_{mi}$

Equivalent *input noise* PSD: $\hat{V}_{neq}^2(f) = \frac{16}{3}\frac{kT}{g_{m1}}\left[1+\frac{g_{m3}}{g_{m1}}\right] \cong \frac{16}{3}\frac{kT}{g_{m1}} \quad V^2/Hz$

for $g_{m1} >> g_{m3}$. It can be represented by a noisy resistor $R_N = (8/3)kT/g_{m1}$ at one input terminal. Choose g_{m1} as large as practical!

All noises thermal - white. Opamp dynamics ignored.

Thermal Op-Amp Noise – (2)

All devices in active region, $[id(f)]^2=(8/3)kT \cdot g_m$. Consider the short-circuit output current $I_{0,sh}$ of the opamp. The output voltage is $I_{o,sh} \cdot R_o$.

If the *i*th device PSD is considered, its contribution to the PSD of $I_{0,sh}$ is proportional to g_{mi}. Referring it to the input voltage, it needs to be divided by the square of the input device g_m, i.e, by g_{m1}^2.

Hence, the input-referred noise PSD is proportional to g_m/g_{m1}^2. For the noise of the input device, this factor becomes $1/g_{m1}$.

Conclusions: Choose input transconductances (g_{m1}) as large as possible. For all non-input devices (loads, current sources, current mirrors, cascade devices) choose g_m as small as possible!

Noisy Op-Amp with Compensation

Op-amp has thermal input noise PSD $P_{ni} = 16kT/3g_{m1}$, where g_{m1} is the transconductance of the input devices.

Single-pole model; voltage gain $A(s) = A_0\omega_{3\text{-}dB}/(s+\omega_{3\text{-}dB})$, where A_0 is the DC gain, $\omega_{3\text{-}dB}$ is the 3-dB BW (pole frequency), and $\omega_u = A_0\omega_{3\text{-}dB}$ is the unity-gain BW of the op-amp. For folded-cascode telescopic and 2-stage OTAs, usually $\omega_u = g_{m1}/C$, where C is the compensation capacitor. *Open-loop* noise BW is $f_n = g_{m1}/4A_0C$, and the open-loop noise gain at DC is A_0. Hence, the *open-loop* output noise power is

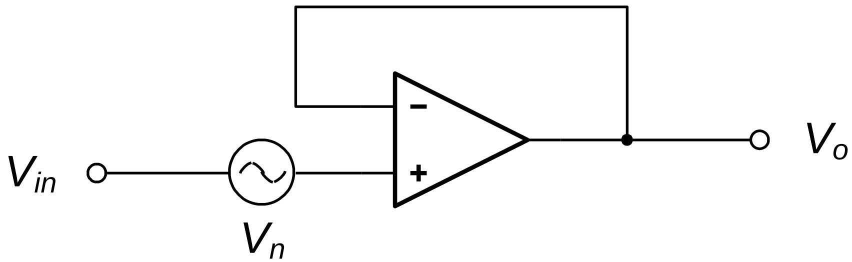
$$P_{on} \cong \frac{16}{3}\frac{kT}{g_{m1}}A_0^2\frac{g_{m1}}{4A_0C} = \frac{4A_0kT}{3C}.$$

Noisy Op-Amp with Unity-Gain Feedback

If the op-amp is in a *unity-gain* configuration, then (for $A_0 >> 1$) the noise bandwidth of the stage becomes $A_0 f_n$, and the DC noise gain is 1. Hence, the output (and input) thermal noise power is

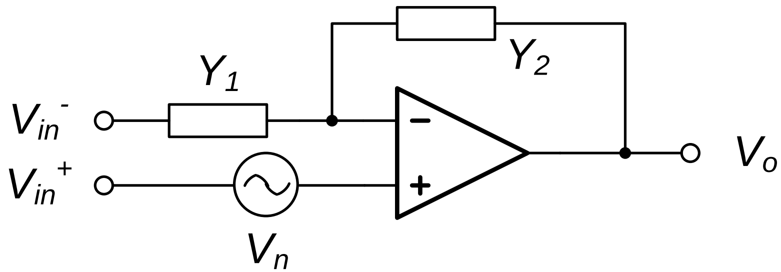
$$P_n \cong \frac{16kT}{3g_{m1}} \frac{g_{m1}}{4C} = \frac{4kT}{3C},$$

(This result is very similar to the ***kT/C*** noise power formula of the simple R-C circuit!)



Noisy Op-Amp in an SC Gain Stage

A more general feedback stage:



Let $G_i = Y_1/Y_2$ be constant. Then the noise voltage gain is the single-pole function $A_n(s) = V_{on}(s)/V_n(s) = \omega_u/(s + \omega'_{3-dB})$ where $\omega'_{3-dB} = \omega_u/(1+G_i)$ is the 3-dB frequency of $A_n(j\omega)$. The DC noise gain is $\omega_u/\omega'_{3-dB} = 1 + G_i$, and the noise BW of the stage is $f'_n = g_{m1}/[4C(1+G_i)]$. Hence, the *output* thermal noise power is

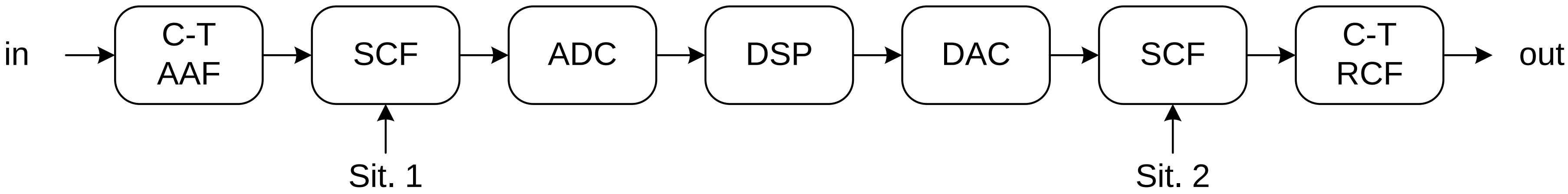
$$P_{no} \cong \frac{16kT}{3g_{m1}}(1+G_i)^2 \frac{g_{m1}/4C}{1+G_i} = \frac{4(1+G_i)kT}{3C},$$

and the *input*-referred thermal noise power is P_{ni} = (4/3) $kT\beta/C$, where $\beta = 1/(G_{i+1})$ is the *feedback factor.*

P_{ni} is smaller for a higher gain G_i, so a higher SNR is possible for higher stage gain.

Switched-Capacitor Noise – (1)

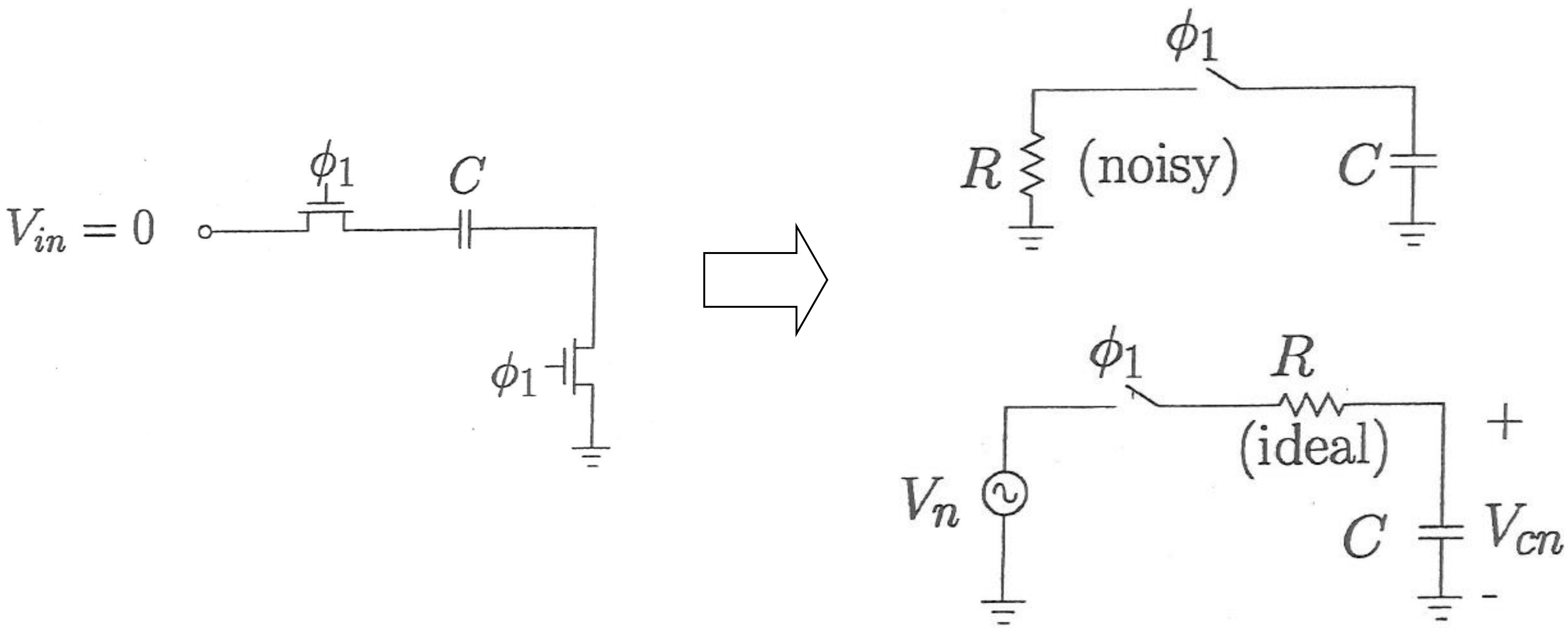
Two situations; example:



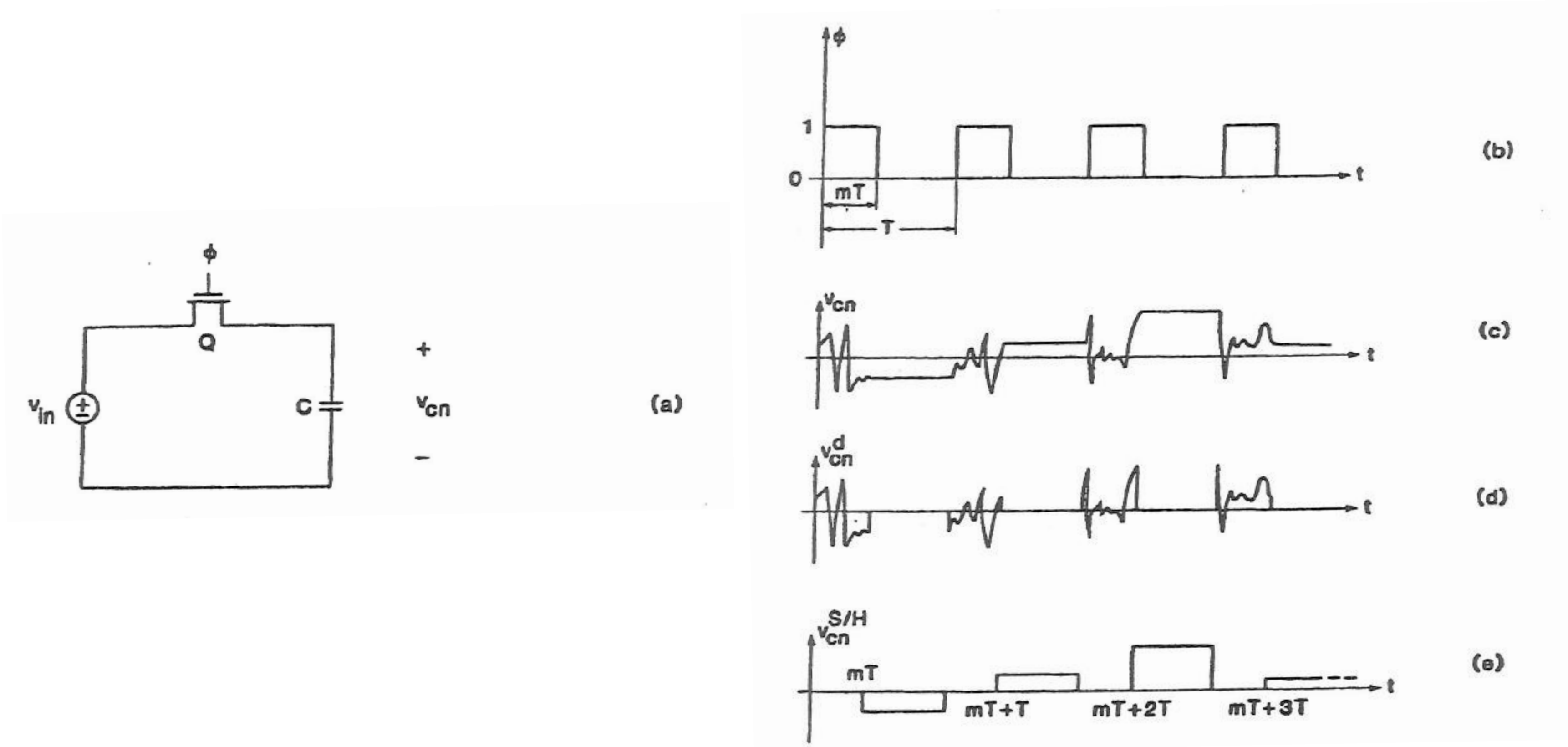
Situation 1: only the sampled values of the output waveform matter; the output spectrum may be limited by the DSP, and hence $V_{RMS,n}$ is reduced. Find V_{RMS} from $\sqrt{kTC}$ charges; adjust for DSP effects. Noise can be estimated by hand analysis.

Situation 2: the complete output waveform affects the SNR, including the S/H and direct noise components. Usually the S/H noise dominates at low frequencies. High-frequency noise is reduced by the reconstruction filter. *Needs CAD analysis.*

Sampling the Noise

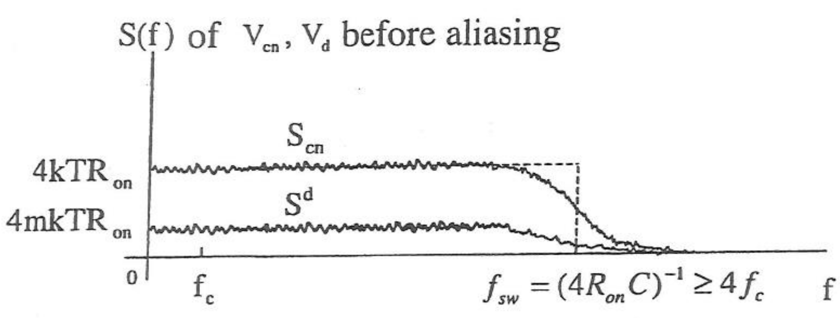


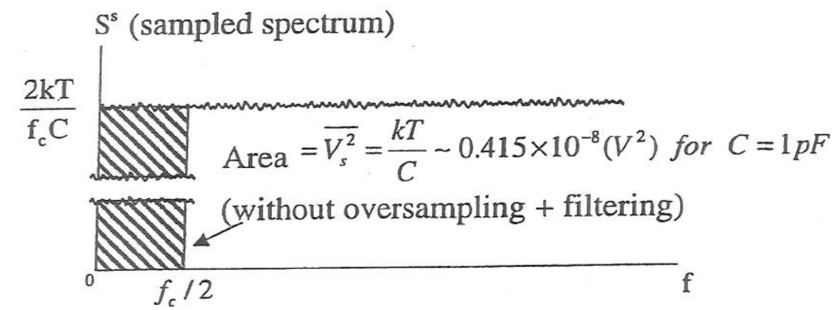
Switched-Capacitor Noise

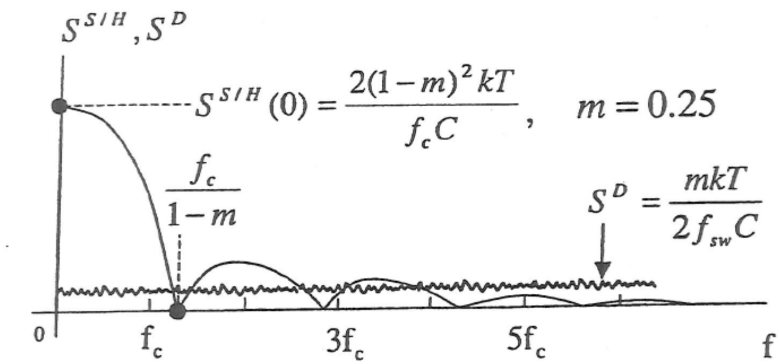


Thermal noise in a switched-capacitor branch: (a) circuit diagram; (b) clock signal; (c) output noise; (d) direct noise component; (e) sampled-and-held noise component. The noise power is kT/C in every time segment.

Switched-Capacitor Noise Spectra







For $f \ll f_c$, $S^{S/H} \gg S^D$. Sampled *PSD* = 2*kT/fsC ;* the unsampled *PSD* = *4kTRon.*

Their ratio is 1/(2*fs.Ron.C) >> 1* !! Sampling penalty.

Switched-Capacitor Noise

The MS value of samples in $\boldsymbol{V_{cn}^{S/H}}$ is unchanged:

$$\overline{\left(V_{cn}^{S/H}\right)^2} = kT/C$$

Regarding it as a continuous-time signal, at low frequency its one-sided PSD is

$$S^{S/H}(f) \cong \frac{2(1-m)^2 kT}{f_c C}$$

while that of the direct noise is

$$S^d(f) \cong \frac{mkT}{f_{sw} C},$$

$$\frac{S^{S/H}}{S^d} = \frac{2(1-m)^2}{m} \frac{f_{sw}}{f_c}.$$

Since we must have $\boldsymbol{f_{sw}/f_c > 2/m}$, usually $|\boldsymbol{S^{S/H}}| >> |\boldsymbol{S^d}|$ at low frequencies. (See also the waveform and spectra.)

See Gregorian-Temes book, pp. 505-510 for derivation.

Calculation of SC Noise – (1)

In the switch-capacitor branch, when the switch is on, the capacitor charge noise is lowpass-filtered by R_{on} and C. The resulting charge noise power in C is kTC. It is a colored noise, with a noise-bandwidth $f_n = 1/(4 \cdot R_{on} \cdot C)$. The low-frequency PSD is $4kTR_{on}$.

When the switch operates at a rate $f_c << f_n$, the samples of the charge noise still have the same power kTC, but the spectrum is now white, with a $PSD = 2kTC/f_c$. For the situation when only discrete samples of the signal and noise are used, this is all that we need to know.

For continuous-time analysis, we need to find the powers and spectra of the direct and S/H components when the switch is active. The direct noise is obtained by windowing the filtered charge noise stored in C with a periodic window containing unit pulses of length m/f_c. This operation (to a good approximation) simply scales the PSD, and hence the noise power, by m. The low-frequency PSD is thus $4mkTR_{on}$. For complex circuits, CAD is required to find noise.

Calculation of SC Noise (summary) – (2)

To find the PSD of the S/H noise, let the noise charge in C be sampled-and-held at fc, and then windowed by a rectangular periodic window

$$w(t)=0 \quad \text{for } n/f_c < t < n/f_c + m/f_c$$
$$w(t)=1 \quad \text{for } n/f_c + m/f_c < t < (n+1)/f_c$$
$$n = 0, 1, 2, \ldots$$

Note that this windowing reduces the noise power by (1 - *m*) squared(!), since the S/H noise is not random within each period.

Usually, at low frequencies the S/H noise dominates, since it has approximately the same average power as the direct noise, but its PSD spectrum is concentrated at low frequencies. As a first estimate, its PSD can be estimated at $2(1-m)^2kT/f_c \cdot C$ for frequencies up to $f_c/2$.

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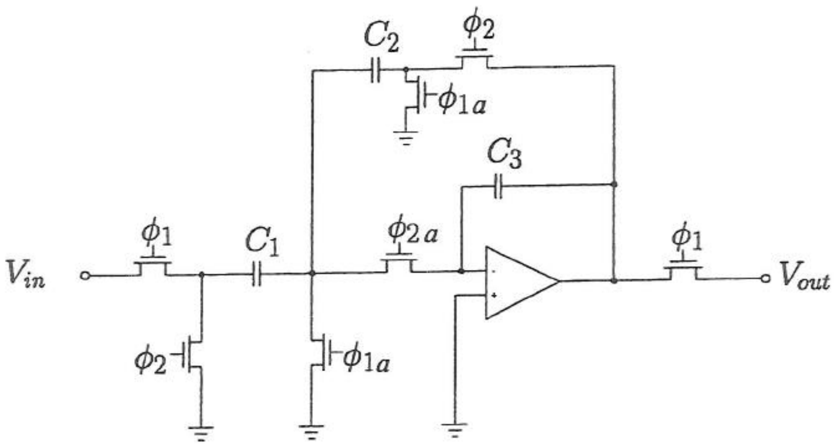
Circuit Example: Lossy Integrator with Ideal Op-Amp[4]



ϕ_{ia}: advanced cutoff phase

RMS noise charges acquired by C_i during $\phi_1 = 1$:

$$q_i^j = C_i\sqrt{kT/C_i} = \sqrt{kTC_i}, i, j = 1,2$$

with V_{in} set to 0.

RMS noise charge delivered into C_3 as $\phi_2 \to 0$, assuming OTA:

From C_1: $$q_1 = \sqrt{2kTC_1}\frac{C_3}{C_2+C_3}$$

Form C_2: $$q_2 = \left[(kTC_2)\left(\frac{C_3}{C_2+C_3}\right)^2 + kT\frac{C_2C_3}{C_2+C_3}\right]^{1/2}$$

Total: $$q_3 = \frac{C_3}{C_2+C_3}\sqrt{kT}\left[2C_1 + 2C_2 + \frac{C_2^2}{C_3}\right]^{1/2}$$

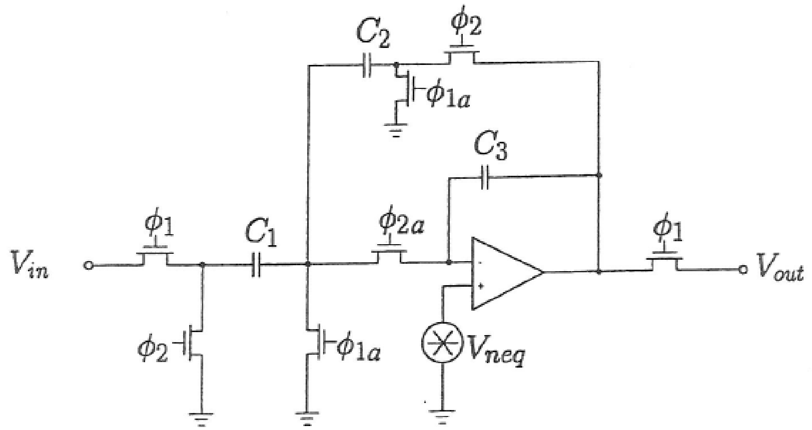
Input-referred RMS noise voltage:

$$V_{in,n} = q_3\frac{C_2+C_3}{C_1C_3} = \frac{\sqrt{kT}}{C_1}\left[2C_1 + 2C_2 + \frac{C_2^2}{C_3}\right]^{1/2}$$

$$V_{in,n} \cong \frac{1}{C_1}\sqrt{kT(C_1+C_2)} \ for \quad C_2 << C_3.$$

$V_{in,n}$ and V_{in} are both low-pass filtered by the stage.

Sampled Op-Amp Noise Example [4]



- ϕ_1=1

Direct noise output voltage = V_{neq}

- ϕ_2=1

Charges delivered by C_1 and C_2:

$$-C_1(V_{neq}+V_{in})+C_2(V_0-V_{neq}).$$

Charge error $-(C_1+C_2)V_{neq}$.

- Input-referred error voltage

$$V_{neq}(1+C_2/C_1).$$

Reference

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